

Quantum abstract neural automata

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Abstract

The nineties of the 20th century, I have proposed and constructed theory of Abstract Neural Automata (ANA) [1]. Aim of present manuscript is exploring theory of ANA on quantum state. Some results have been obtained.

Keywords : Gibbs measure , Quantum abstract neural automata, Existence , von Neumann algebra , Isomorphic , Consciousness.

1. Introduction

The nineties of the 20th century, I have proposed and constructed theory of Abstract Neural Automata (ANA) [1]. Some of the work having done:

Abstract neural automata are established on N-dimensional lattices、 Euclidean Spaces and compact Riemannian manifolds respectively; and the objectives are achieved: concept generation, concept connection transfer and thinking formation in abstract neural automata; The origin of thought; There is a positive correlation between brain intelligence and the volume of related brain regions; The ergodicity of the genetic evolution of human brain intelligence; Abstract neural automata has a unique solution if and only if it goes to infinity in both micro and macro; (Consciousness is expected to unify micro and macro!); Equivalent proposition: Abstract neural automata is Markov; The limiting probability distribution of abstract neural automata is Gibbsian; Abstract neural automata are evolutionary; Assuming the existence of information on network of the stochastic automata, abstract neural automata are forever

moving machines in the sense of evolution; The representation theory of knowledge by abstract neural automata is established.

Aim of present manuscript is exploring theory of ANA on quantum state(QANA).The precondition for writing this article is to make sure that quantum mechanics plays a role in brain function[2][3][4].In particular, Consciousness depends on quantum states of electrons within hydrophobic pockets in a class of brain proteins.

Proof of existence of QANA on the von Neumann algebra $L^\infty(X, \mu)$ and on the C*—

algebra \mathcal{A} has been given in present article; For $\Phi \in \mathcal{B}_1$ and $\psi \in I(\tau)$,

following proposition is mutually equivalent:

(1)There exists QANA on C*-algebra \mathcal{A} .

(2) ψ is (σ^Φ, β) -KMS state.

(3) $P(\psi) = S(\psi) - \beta\psi(A_\Phi)$;

Conditional state of quantum measurement is Gibbsian; the brain of the observer begin the process of experiencing consciousness in process of the measurement . In particular, it is quite natural for us to see from the point of view of physics and mathematics that consciousness is the collapse of quantum states in the human brain system at the microscopic level, and is Gibbsian.

2.Definitions and a theorem about Abstract neural automata[1]

Definition 1.A stochastic automaton a is defined as triplet with a function—pair

$$a=((Q,Y,Z),(F,G)) \quad (1)$$

where Q :space of state; Y : set of input; Z :set of output; $F:Y \times Q^2 \rightarrow [0, 1]$, such that

$$\forall y \in Y, \forall q \in Q,$$

$$\sum_{q^* \in Q} F(y, q, q^*) = 1 \quad (2)$$

exists; $G:Q \times Z \rightarrow [0, 1]$,such that $\forall q \in Q,$

$$\sum_{z \in Z} G(q, z) = 1 \quad (3)$$

exists.

In fact, $F(y, q, q^*)$:the transition probability from state q to q^* when input is y ;

$G(q, z)$: probability for output to be z when state is q .

Definition 2. A network S of stochastic automaton \mathbf{a} is defined as a pair consisted of d -dimensional the integer lattices(d -dimensional neuron -set T^d) and a function-pair (R, C)

$$S = (T^d, (R, C)) \quad (4)$$

where $R: T^d \rightarrow A$, A is a set of all automata which is defined as (Q, Y, Z) ; $C: T^d \times Z \rightarrow Y$; in fact, $R(t)$ is an automaton \mathbf{a} which corresponds to $t \in T^d$; C is a connected transformation of some network $N \subset S : \forall f \in Z^{T^d}, t_0 \in T^d$, when neuron $t \in T^d$ gives a output $f(t)$, $C(t_0, f) = y$ is an input which is received by t_0 .

Definition 3. Abstract Neural Automata(ANA) which are on the network is a set of random element $\{\xi(t), t \in T^d\}$, $\xi(t) : (A, B(A), P) \rightarrow (Q^{T^d} = X, B(X), \mu)$, where $(A, B(A), P)$ and $(Q^{T^d} = X, B(X), \mu)$ are two space of probability.

Definition 4. To every finite subset $\tilde{L} = (t_1, \dots, t_n) \subset T^d$ there corresponds probability distribution $\mu(x_{t_i}, x_{t_i} \in Q, i = 1, \dots, n)$, such that $\mu(\xi_{t_i} = x_{t_i}, i = 1, \dots, n) = \mu_L(x_{t_i}, x_{t_i} \in Q, i = 1, \dots, n)$ and satisfies consistency condition with $L = (t_1, \dots, t_m) \subset \tilde{L}$,

$$\sum_{x_{t_{m+1}} \in Q, \dots, x_{t_n} \in Q} \mu_L(x_{t_i}, i = 1, \dots, n) = \mu_L(x_{t_i}, i = 1, \dots, m) \quad (5)$$

We refer to the family of probability distribution $\mu = \{\mu_L(\bullet)\}$ as probability distribution of ANA.

Definition 5. Family of conditional probability distribution

$$\Lambda \{ \lambda_L(x_{t_i}, i = 1, \dots, n) \mid x(t), t \in T^d \setminus \tilde{L} \} \quad (6)$$

Is called a conditional distribution of ANA, where $\mu\{(\xi(t_i) = x_{t_i}, i = 1, \dots, n) \mid \xi(t), t \in T^d \setminus \tilde{L}\} = \lambda_L((x_{t_i}, i = 1, \dots, n) \mid x(t), t \in T^d \setminus \tilde{L})$ with probability one.

(7)

Definition 6. When $L = (t_1, \dots, t_m) \subset T^d$, for arbitrary $x_1 \in Q, \dots, x_m \in Q$, and for arbitrary function $x(t), t \in T^d \setminus L$, conditional probability distribution

$$\begin{aligned} \lambda_L((x_1, \dots, x_m) | x(t)) \\ = \frac{1}{\Phi_L(x(t))} \exp\{-\beta U_L((x_1, \dots, x_m) | x(t))\} \end{aligned} \quad (8)$$

Is called Gibbs distribution with a boundary condition $x(t)$ in L .

Where

$$\Phi_L(x(t)) = \sum_{x_1 \in Q, \dots, x_m \in Q} \exp\{-\beta U_L((x_1, \dots, x_m) | x(t))\} \quad (9)$$

And β constant, $0 < \beta < \infty$, inversely proportional to the temperature of network

S of stochastic automata; U_L is called potential

$$\begin{aligned} U_L((x_1, \dots, x_m) | x(t)) \\ = -c \sum_{i=1}^m x_i + \frac{1}{2} \sum_{i=1}^m \sum_{j=1, j \neq i}^m x_i x_j U(t_i - t_j) + \sum_{i=1}^m \sum_{(T^d - L)} x_i x(t) U(t_i - t) \end{aligned} \quad (10)$$

Where c is a constant, the chemical potential.

Definition 7. ANA and its distribution are called Gibbs, if its conditional

Distribution is given by equation (8).

Theorem 1. In theory of ANA, suppose collection of potential-functions of various form is $G(U)$, and that Q is compact metric space, then for given Continuous potential function $G(U)$, ANA there exists on probability

Space $(Q^{T^d} = X, B(X), \mu)$ [1].

3. Quantum abstract neural automata (QANA)

To describe abstract neural automata at quantum level (QANA), we associate with each neuron $a \in T^d$, quantum automaton, a finite dimensional Hilbert space

\mathcal{H}_a . Hence, we have Hilbert space $\mathcal{H}_\Lambda = \bigotimes_{a \in \Lambda \subset T^d} \mathcal{H}_a$ and then local C*-algebra

$\mathcal{A}_\Lambda = \bigotimes_{a \in \Lambda \subset T^d} \mathcal{A}_a$, if C*-algebra $\mathcal{A}_a = B(\mathcal{H}_a)$.

Next we put

$$\mathcal{S} = \{\Lambda \subset T^d, 0 < |\Lambda| < \infty\}$$

where $|\Lambda|$ is the cardinality of set Λ . Note that \mathcal{S} is countable. On family of set

$\{\mathcal{A}_\Lambda, \Lambda \in \mathcal{S}\}$, let $\Lambda_2 \setminus \Lambda_1 = \{k \in \Lambda_2, k \notin \Lambda_1\}$, then

$\mathcal{A}_{\Lambda_2} = \mathcal{A}_{\Lambda_1} \otimes \mathcal{A}_{\Lambda_2 \setminus \Lambda_1}$, further when $\Lambda_1 \subset \Lambda_2$, there exists embedding $j_{\Lambda_2 \Lambda_1}$ from \mathcal{A}_{Λ_1} to \mathcal{A}_{Λ_2} , defined by $j_{\Lambda_2 \Lambda_1}(A) = A \otimes I_{\Lambda_2 \setminus \Lambda_1}$ ($\forall A \in \mathcal{A}_{\Lambda_1}$, I_Λ , identity operator of \mathcal{A}_Λ), having following property

$$(1) \Lambda_1 \subset \Lambda_2 \subset \Lambda_3 \Rightarrow J_{\Lambda_3 \Lambda_1} = J_{\Lambda_3 \Lambda_2} J_{\Lambda_2 \Lambda_1};$$

$$(2) \Lambda_1 \subset \Lambda, \Lambda_2 \subset \Lambda, \Lambda_1 \cap \Lambda_2 = \emptyset$$

$$\Rightarrow [J_{\Lambda \Lambda_1}(A_1), J_{\Lambda \Lambda_2}(A_2)] = 0, \forall A_1 \in \mathcal{A}_{\Lambda_1} \\ \forall A_2 \in \mathcal{A}_{\Lambda_2}.$$

Therefore, when $\Lambda_1 \cap \Lambda_2 = \emptyset$, we have $\mathcal{A}_{\Lambda_1} = \mathcal{A}_1 \otimes I_{\Lambda_2}$, hence $\{\mathcal{A}_\Lambda, \Lambda \in \mathcal{S}\}$ is increasing family. And then, $\mathcal{A}_0 = \bigcup_{\Lambda \in \mathcal{S}} \mathcal{A}_\Lambda$ is normed \ast -algebra, completion of \mathcal{A}_0 by means of this norm $\mathcal{A} = \overline{\bigcup \{\mathcal{A}_\Lambda; \Lambda \in \mathcal{S}\}}$ is known as local C^* -algebra of local observables.

For $a = (a_1, \dots, a_d) \in T^d$, we define $\Lambda(a) \subset T^d$ as $\Lambda(a) = \{b \in T^d; 0 \leq b_j \leq a_j, j = 1, \dots, d\}$. The set of translation of $\Lambda(a)$ by $na = (n_1 a_1, \dots, n_d a_d)$

$$\Lambda_n = \Lambda(a) + na, \quad n \in T^d$$

form a partition of T^d . For any $\Lambda \subset T^d$, we let $N_a^+(\Lambda)$ (resp. N_a^-) to be the number of $\Lambda_n \cap \Lambda \neq \emptyset$ (resp. $\Lambda_n \subset \Lambda$) of Λ_n . At this time, finite subset

$\Lambda \subset T^d$ are defined to tend to ∞ in the sense of Van Hove, if

$$\lim_{\Lambda \rightarrow \infty} N_a^-(\Lambda) / N_a^+(\Lambda) = 1, \quad (11)$$

to express as $\nearrow \infty$.

For arbitrary $a, \xi \in T^d$, suppose unitary mapping $U_\xi(a)$ from \mathcal{H}_ξ to $\mathcal{H}_{\xi+a}$ satisfies conditions (1) $U_\xi(0) = I_\xi$, (2) $U_\xi(a+b) = U_{\xi+b}(a)U_\xi(b)$ ($\forall b \in T^d$); and

let $U_\Lambda(a) = \bigotimes_{\xi \in \Lambda} U_\xi(a)$. Using this unitary operator, let

$$\tau_a(\Lambda) = U_\Lambda(a) \Lambda U_{\Lambda+a}(-a). \quad (12)$$

Hence τ is automorphism mapping from \mathcal{A}_Λ to $\mathcal{A}_{\Lambda+a}$ (i.e., $\tau: T^d \rightarrow \text{Aut}(\mathcal{A})$).

Suppose φ is the state on \mathcal{A} . Restriction φ_Λ of φ to \mathcal{A}_Λ is known as local normal state if it satisfies condition: there exists $\rho_\Lambda \in T(\mathcal{H}_\Lambda)_{+,1}$, $\varphi_\Lambda(A) = \text{tr}_\Lambda \rho_\Lambda A$ ($\forall A \in \mathcal{A}_\Lambda$), hence we have $\varphi = \{\rho_\Lambda\}$ when $\rho_\Lambda = \text{tr}_{\Lambda \setminus \Lambda'} \rho_{\Lambda'}$ ($\Lambda \subset \Lambda'$). Moreover if $\varphi = \{\rho_\Lambda\}$ satisfies condition $\rho_{\Lambda+a} = U_\Lambda(a) \rho_\Lambda U_{\Lambda+a}(-a)$, then φ is known as τ -invariant (i.e., $\varphi \in I(\tau)$) or T^d -invariant.

For $\varphi = \{\rho_\Lambda\}$, entropy of ρ_Λ equals $S_\Lambda(\varphi) = -\text{tr}_\Lambda \rho_\Lambda \log \rho_\Lambda$.

Theorem 2. Let $\varphi \in I(\tau)$, and suppose $|\Lambda|$ is volume of Λ , then we have:

$$(1) S(\varphi) = \lim_{|\Lambda| \rightarrow \infty} \frac{1}{|\Lambda|} S_\Lambda(\varphi), 0 \leq S(\varphi) \leq \log 2. \quad (13)$$

$$(2) S(\varphi) \text{ is affine and upper semi-continuous function on } I(\tau).$$

If mapping $\Phi: \Lambda \in \mathcal{S} \rightarrow \Phi(\Lambda) \in \mathcal{A}_\Lambda$ satisfies condition

$\Phi(\Lambda) = \Phi(\Lambda)^*$, then Φ is called interaction potential (or simply a potential).

Let $\|\Phi\| = \sum_{\Lambda \ni 0} \frac{\|\Phi(\Lambda)\|}{|\Lambda|}$, set of all Φ ($\|\Phi\| < \infty$) be \mathcal{B} , then, \mathcal{B} becomes

real Banach space for norm $\|\Phi\|$. Further, Φ is known as finite range if number of

finite set Λ to have made $\Phi(\Lambda) \neq 0$ ($\Lambda \ni 0$) is finite. Let all of interaction

potentials $\Phi \in \mathcal{B}$ of finite range be \mathcal{B}_0 , hence completion of \mathcal{B}_0 for norm

$\|\cdot\|$ equals \mathcal{B} , to express as $\overline{\mathcal{B}_0} = \mathcal{B}$. When $\Phi(\Lambda+a) = \tau_a(\Phi(\Lambda))$, Φ is

known as T^d -invariable or translationally invariable; there exists some number n_0 , when $|\Lambda| > n_0$, $\Phi(\Lambda) = 0$, Φ is known as interaction potential of finite body.

For $\Phi \in \mathcal{B}$, let

$$H_\Lambda(\Phi) = \sum_{X \subset \Lambda} \Phi(X) \quad (14)$$

and

$$A_\Phi = \sum_{\emptyset \in \Lambda} \frac{\Phi(\Lambda)}{|\Lambda|} \quad (15),$$

then for $\varphi \in I(\tau)$, from $|\Lambda|^{-1} \varphi(\sum_{a \in \Lambda} \tau_a(A_\Phi)) = \varphi(A_\Phi)$, we have

$\lim_{\Lambda \nearrow \infty} |\Lambda|^{-1} \varphi(H_\Phi(\Lambda)) = \varphi(A_\Phi)$. For thermodynamic functions

$$Z_\Lambda(\Phi) = \text{tr}_\Lambda e^{-H_\Lambda(\Phi)}, \quad (16)$$

And

$$P_\Lambda(\Phi) = \frac{1}{|\Lambda|} \log Z_\Lambda(\Phi), \quad (17)$$

We have

Theorem 3. Suppose $\Phi \in \mathcal{B}$ is T^d -invariant, then

(1) there exists $P(\Phi) = \lim_{\Lambda \nearrow \infty} P_\Lambda(\Phi)$.

(2) $P(\Phi) = \sup\{S(\psi) - \psi(A_\Phi); \psi \in I(\tau)\}$.

(3) $P(\Phi)$ is continuous convex function, and

$$|P(\Phi) - P(\Psi)| \leq \|\Phi - \Psi\|, \forall \Phi, \Psi \in \mathcal{B}.$$

Lemma 1. Suppose that M is a commutative von Neumann algebra, then there exists locally compact Hausdorff space E which satisfies second axiom of countability, and measure ν , such that $M = L^\infty(E, \nu)$.

Lemma 2. A C^* -algebra \mathcal{A} is a von Neumann algebra if and only if it has dual \mathcal{A}^* as a Banach space.

Denote set of linear functional φ from C^* —algebra \mathcal{A} to \mathbf{C} (complex field) by \mathcal{A}^* , since C^* —algebra \mathcal{A} is bounded normed linear space, its dual space is Banach space for norm $\varphi(A) = \|\varphi\|, A \in \mathcal{A}$. When $\|\varphi\| = 1$, φ is called state on \mathcal{A} . \mathcal{A}^* is called space of state. From Lemma 2, C^* —algebra \mathcal{A} is von Neumann algebra. Hence from Lemma 1 and Lemma 2, we have following

Theorem 4. Let $L^\infty(E, \nu) = L^\infty(X, \mu)$, hence there exists QANA on von Neumann algebra $L^\infty(X, \mu)$.

Proof. Let $L^\infty(X, \mu) = (E, \mathcal{E}, \nu)$, then, from the nature of $L^\infty(X, \mu)$, we have

$$\nu_f(B, B \in \mathcal{E}) = \mu(f^{-1}(B)) = \mu\{x : f(x) \in B\}, a.e.,$$

therefore $L^\infty(X, \mu)$ and (X, μ) are isomorphic. Hence there exists ANA on (X, μ)

(Theorem 1) \Leftrightarrow there exists QANA on the $L^\infty(X, \mu)$. QANA =

$$C(\rho|P_a) = \rho = \frac{\rho}{\text{Tr}(\rho)} = \frac{e^{-\beta H}}{\text{Tr}(e^{-\beta H})} \quad (23) \quad (\text{see subsection 4}).$$

Suppose set of all potentials $\Phi \in \mathcal{B}_0$ which are T^d -invariant and have interaction of

finite body is \mathcal{B}_1 . For $\Phi \in \mathcal{B}_1$, there exists $\sigma_t^\Phi \in \text{Aut}(\mathcal{A})$ ($\forall t \in \mathbb{R}$) (i.e.,

$$\lim_{\Lambda \uparrow T^d} \|\sigma_t^\Phi(A) - e^{itH_\Lambda(\Phi)} A e^{-itH_\Lambda(\Phi)}\| = 0, \forall A \in \mathcal{A}.$$

Definition 8. Let σ_t be a strongly—continuous one—

parameter group of automorphism of \mathcal{A} ; $\beta \in \mathbb{R}$ (real number). One says that a state

$\psi \in \mathcal{A}$ is KMS_β state for σ if it is invariant under σ_t , i.e. $\psi \in I(\tau)$, and

for all $a, b \in \mathcal{A}$ there exists a function $F_{a,b}(z)$ bounded and continuous on the

strip $0 \leq \text{Im}(z) \leq \beta$ and analytic on $0 < \text{Im}(z) < \beta$ such that

$$F_{a,b}(t) = \psi(a\sigma_t(b)) \text{ for all } t \in \mathbb{R}; \text{ and } F_{a,b}(t+i\beta) = \psi(\sigma_t(b)a) \text{ for all } t \in \mathbb{R}.$$

Proposition. Let $A = K(\mathcal{H})$ be the algebra of compact operators on a Hilbert

space \mathcal{H} , $\beta \in \mathbb{R}$ (real number) and H a self—

Adjoint operator such that $e^{-\beta H}$ is trace class. Then the Gibbs state with density matrix

$\varphi = \frac{e^{-\beta H}}{\text{Tr}(e^{-\beta H})}$ is the unique KMS—state for the one —parameter group generated by H .

Theorem 5. For $\Phi \in \mathcal{R}_1$ and $\psi \in I(\tau)$, following proposition is mutually equivalent:

(1) There exists QANA on C*-algebra \mathcal{A} .

(2) ψ is KMS_β state.

(3) $P(\psi) = S(\psi) - \beta \psi(A_\Phi)$. [5]

4.Coexistence of N and Q

The content of this section is to explain the coexistence of QANA and ANA in the brain and their interdependent relationship by means of the mean of method of text [6].

Suppose that human neural system at the ion level (QANA) is denoted by Q, while the human neural system at level of neural network(ANA) is denoted by N, that the category von Neumann algebra of N is

$$A = C^A$$

for some finite set A , that the category von Neumann algebra of Q is B . Let's do linear map

$$\varepsilon: B^* \rightarrow A^*.$$

Hence we have

$$\varepsilon(\rho) = \bigoplus_{a \in A} \rho(E_a), \quad (18)$$

where $\text{POVM } E_a \geq 0$, and

$$\sum_{a \in A} (E_a) = 1. \quad (19)$$

In fact, $E_a: A \rightarrow \mathcal{A}$, a positive operator-valued measure on A , POVM, that takes values in positive cone \mathcal{A} .

A conditional state on \mathcal{B} :

$$F = \bigoplus_{a \in A} [a] \otimes \rho^{\sqrt{E_a}}. \quad (20)$$

The conditional state of the POVM ε applied to ρ with outcome a is

$$C(\rho|\varepsilon=a)=\rho^{\sqrt{E_a}}, \quad (21)$$

In fact ,we can see the linear map ε as a set-valued measurement which is represented as an orthogonal decomposition:

$$\mathcal{H}=\bigoplus_{a\in A}\mathcal{H}(a)$$

The probability of the outcome a is given by

$$Tr(\rho P_a)$$

And the conditional state of B on A

$$C(\rho|P_a)=\frac{P_a\rho P_a}{Tr(\rho P_a)}$$

Since $\mathcal{H}(a)$ is invariant subspace of ρ ,thus

$$C(\rho|P_a)=\frac{\rho P_a}{Tr(\rho P_a)}=\frac{\rho E_a}{Tr(\rho E_a)}. \quad (22)$$

The process of a measurement is process of the decoherence.State in that the decoherence has been achieved is called trivial quantum state—equilibrium state —

classical state. At this time, $\because P_a X = X, \forall X \in \mathcal{H}(a)$,

$\therefore P_a = I$ (trivial projection); and $\because \rho \in T(\mathcal{H})_{+,1}$, $e^{-\beta H} \in T(\mathcal{H})$

$(\Phi(\Lambda)=\Phi(\Lambda)^*)$, \therefore let $\rho=e^{-\beta H}$,hence we have

$$C(\rho|P_a)=\rho=\frac{\rho}{Tr(\rho)}=\frac{e^{-\beta H}}{Tr(e^{-\beta H})} \quad (23)$$

Therefore the trivial conditional state is Gibbsian;conditioning on a measurement is also called “state collapse”or“wave function collapse”; the brain of the observer begin

the process of experiencing consciousness in process of measurement [7].

In particular, it is proved mathematically that the trivial conditional state of quantum measurement is Gibbsian, thus proving that the consciousness of the observer is Gibbsian, which is consistent with the discussion on consciousness in article [7].

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